

# Bias-Tuned Injection-Locked Discriminators

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**Abstract**—A new form of microwave FM discriminator using simultaneously the technique of injection synchronization and automatic bias tuning has been proposed. The present discriminator basically consists of an injection-synchronized Gunn oscillator and an arrangement for bias tuning of the oscillator in correspondence to a measure of the instantaneous phase/frequency error between the incoming signal and the Gunn oscillator output. A detailed analysis on the discriminator performance, which closely fits in with the experimental results, has been presented. The performance of the present discriminator is much better than that of the conventional one.

## I. INTRODUCTION

THOUGH THE IDEA of simultaneous use of direct and indirect synchronization for tracking an RF signal is not new [1], nothing has been reported on its judicious utilization for the purpose of designing an FM discriminator at microwave frequencies without taking recourse to down-converting the frequency of the incoming signal. Based on this principle, we describe a new form of microwave discriminator called a bias-tuned injection-locked discriminator (DILBIT). In its simplest construction it requires one directional coupler, one magic tee, two matched detectors, an adder, a tracking bias source, and a Gunn oscillator. The block diagrammatic representation of the DILBIT is shown in Fig. 1. By referring to the diagram, one can easily appreciate that it basically consists of an injection-synchronized Gunn oscillator, a balanced mixer, and an arrangement for controlling the bias supply of the oscillator. The balanced mixer produces an output proportional to the instantaneous frequency or phase difference between the output of the oscillator  $B \cos(\omega_1 t + \Psi(t))$  and an appropriate fraction of the incoming signal, say  $E_1 \sin(\omega_1 t + \theta(t))$ . Finally, the output of the balanced mixer introduces an additional frequency modulation of the Gunn oscillator over and above that caused through injection synchronization.

## II. PRINCIPLE OF OPERATION

To understand the principle of operation, assume that the oscillator is locked to the incoming signal. Now, if there is a drift in the instantaneous frequency, this will be felt as a phase difference. As a result, the regenerative gain parameter—both the in-phase and quadrature components—will suffer modulations. Further, due to the phase dif-

ference, the balanced mixer will give an output that causes bias modulation, leading to bias tuning of the oscillator. If the polarity of this control voltage is properly chosen, this will try to arrest the attempted drift of the oscillator.

Moreover, the change of the quadrature component of the gain parameter due to injection of the RF input will also arrest the drift of the oscillator frequency. Thus, this particular arrangement gives dual control—one through injection of the RF power to the oscillator and the other through the bias tuning manifested in correspondence to a measure of the instantaneous frequency or phase difference between the two oscillations, appearing at the output of the balanced mixer.

At this point it is important to note that the operation of the system hinges on the performance of the injection-synchronized oscillator (ISO). As such, in the following we briefly scrutinize the performance of the bias-tuned injection-synchronized oscillator.

To proceed further with the analysis, we recognize that  $\Psi(t)$ , the modulated phase of the oscillator, consists of two components:  $\Psi_1(t)$  and  $\Psi_2(t)$ —one due to the RF input and the other due to bias tuning. Now referring to the equivalent circuit of the oscillator [2] as inset in Fig. 1, we assume the circulating charge to be  $q$  and write the oscillator equation as

$$L \frac{d^2 q}{dt^2} + (R + R_L) \frac{dq}{dt} + \frac{q}{C} + v_r + v_c + v_s = 0 \quad (1)$$

where  $L$ ,  $R$ , and  $C$  are, respectively, the equivalent inductance, resistance, and capacitance of the cavity, and  $R_L$  is the load applied to the output terminals of the ISO. The variables  $v_r$  and  $v_c$  denote, respectively, the voltage drops across the resistive and reactive components of the device impedance and are usually assumed to depend on  $q$  as

$$v_r = -\beta_1 \dot{q} + \beta_2 \dot{q}^2 + \beta_3 \dot{q}^3 \quad (2)$$

and

$$v_c = \alpha_1 q + \alpha_2 q^2 + \alpha_3 q^3. \quad (3)$$

Here, the  $\beta$ 's and  $\alpha$ 's are constants of nonlinearities for the device impedance. Further  $v_s$ , the synchronizing signal, is taken as

$$v_s = E \sin(\omega_1 t + \theta(t)). \quad (4)$$

Assuming the solution of (1) to be of the form

$$q = A(t) \cos(\omega_1 t + \Psi(t)) \quad (5)$$

and applying the method of harmonic balance, it is easily

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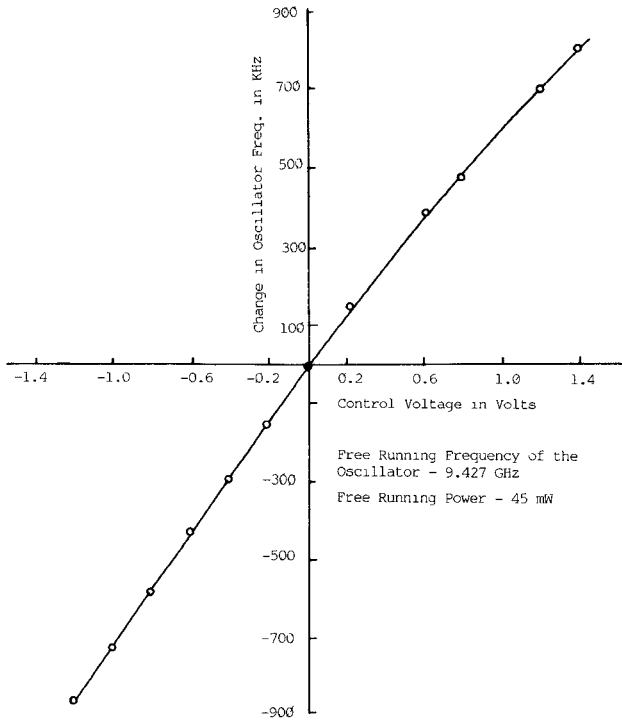


Fig. 2. The variation of the oscillator frequency with the change of the bias voltage.

Now, combining (7), (8), and (11), one can write the governing equations of the system as

$$\frac{da}{dt} = \frac{\omega_o}{2q_1}(1-a^2)a + \frac{\omega_o}{2q_1x}F\cos(\theta - \Psi) \quad (12)$$

and

$$\frac{d\Psi}{dt} = -\Omega + \frac{\omega_o}{2q_1x}\frac{F}{a}\sin(\theta - \Psi) + \gamma k_2 E_1 A(t)\sin(\theta - \Psi) \quad (13)$$

i.e.,

$$\frac{d\Psi}{dt} = -\Omega + \frac{\omega_o}{2q_1x}\frac{F}{a}(1+ka^2)\sin(\theta - \Psi) \quad (13')$$

where

$$k = 2\gamma k_2 x A_o^2 Q_L R_L. \quad (14)$$

Further, one should note that  $\Omega$  very nearly denotes the detuning of the oscillator with respect to the synchronizing input.

### III. STABLE ZONES OF FREQUENCY RESPONSE CHARACTERISTICS OF THE OSCILLATOR

In this section we consider the response characteristics of the locked oscillator for (1) low strength of the synchronizing signal and (2) moderate strength of the synchronizing signal (cf. Fig. 1). We deliberately avoid the case of the overdriven situation, because it does not appear in practical cases.

#### A. Low-Level Injection Locking

For low values of the strength of the synchronizing signal, the oscillator amplitude does not vary with the detuning under locked condition, i.e., when  $\dot{a} = 0$ ,  $\dot{\Psi} = 0$ , and  $a_s = 1.0$ . In this case, the locking range of the oscillator is given by (13'):

$$\Omega = \pm \frac{\omega_o}{2q_1x}F(1+k).$$

The variation of the locking range with the strength of the incoming signal is shown in Fig. 3, obtained experimentally on the model of Fig. 1. This figure clearly indicates the enhancement of the locking range through automatic bias tuning.

#### B. Moderate-Level Injection Locking

For a synchronizing signal, of moderate strength, denoting the steady-state values of the amplitude and the phase error as  $a_s$  and  $\Psi_s$ , respectively, the frequency response characteristic can easily be shown to be

$$(1-a_s^2)^2 + \left( \frac{2q_1\Omega}{\omega_o(1+ka_s^2)} \right)^2 = \left( \frac{F}{a_s} \right)^2. \quad (15)$$

Again, from (13) it is seen that the condition of locking is

$$\Omega \leq \frac{\omega_o}{2q_1x}\frac{F}{a_s}(1+ka_s^2). \quad (16)$$

Equation (15) is a fifth-order polynomial in  $a_s^2$ ; obviously, a positive real solution of  $a_s^2$  indicates the possibility of entrainment. The variation of  $a_s^2$  with  $\Omega$  is shown in Fig. 4.

Though real values of  $a_s^2$  indicate the possibility of entrainment, this it does not guarantee a stable mode of operation. In order to find this, we adopt the method of Lyapunov [3] and derive the characteristic equation of the system as

$$s^2 + m_1s + m_2 = 0 \quad (17)$$

whence,  $m_1$  and  $m_2$  are given by

$$m_1 = \frac{\omega_o}{2q_1}[(1+ka_s^2)(a_s^2-1) + (3a_s^2-1)] \quad (18)$$

and

$$m_2 = \frac{(1-ka_s^2)\Omega^2}{(1+ka_s^2)^2} + \left( \frac{\omega_o}{2q_1} \right)^2 (3a_s^2-1)(a_s^2-1)(1+ka_s^2). \quad (19)$$

Hence, for stability it is seen that

$$a_s^2 \geq \frac{(k-4) \pm (16.0+k^2)^{1/2}}{2k} \quad (20)$$

and

$$\left( \frac{\omega_o}{2q_1} \right)^2 (3a_s^2-1)(1+ka_s^2)(a_s^2-1) + \frac{(1-ka_s^2)}{(1+ka_s^2)^2}\Omega \geq 0. \quad (21)$$

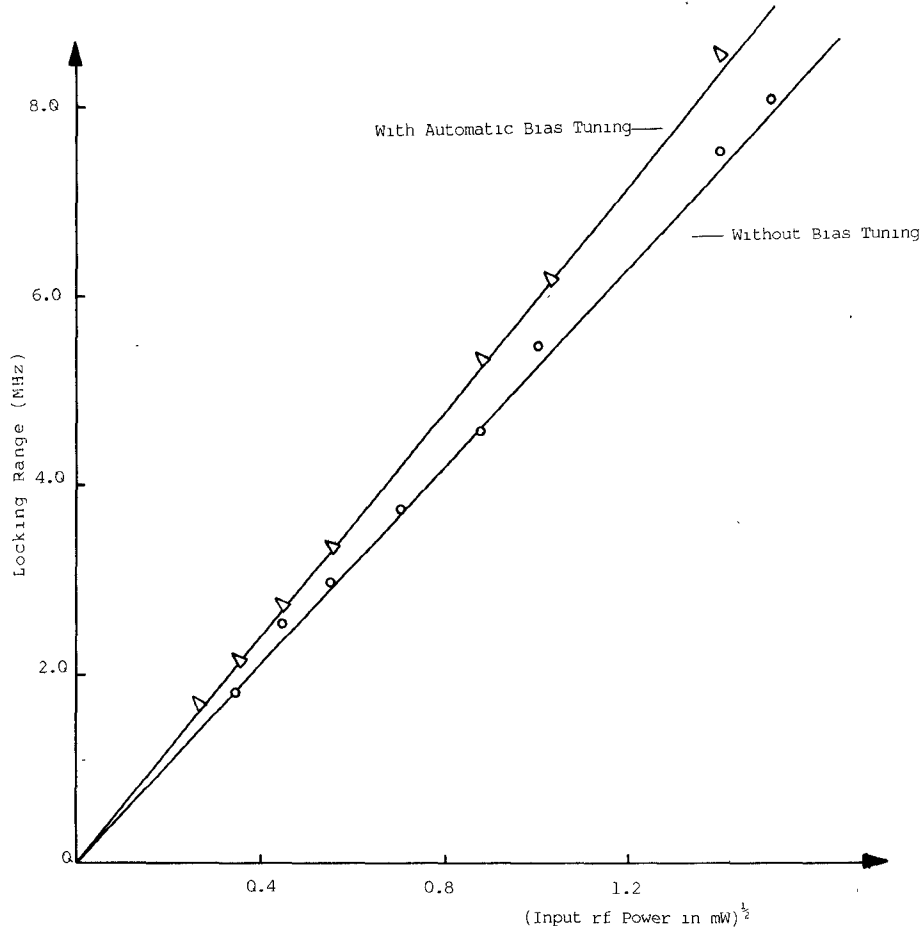


Fig. 3. The variation of the locking range with the strength of the incoming signal.

The stable boundaries obtained from (20) and (21) are superimposed on the frequency response characteristics (cf. Fig. 4). It is interesting and worthwhile to note that the additional tuning control of the oscillator obtained through bias variation improves the stability of the oscillator. This is because a larger value of  $k$  helps avoid the zones of the response characteristics prone to hysteresis and jump phenomena. Incidentally it may be noted that the hysteresis and jump phenomena appear because of the presence of multivalued regions (as shown in Fig. 4) separated by unstable zones. Therefore, referring to Fig. 4, it is readily seen that the method of dual control as proposed in the text improves stability of the system.

#### IV. DISCRIMINATOR CHARACTERISTICS

##### A. Steady State

Refer to (12) and (13') and assume that the input modulation is absent; instead we assume a carrier offset of  $\Omega$ . Further, if the system satisfies the locking condition, the mixer output is written as [cf. (11)]

$$V = -k_2 A_o E_1 a_s \sin \Psi \quad (22)$$

where

$$\sin \Psi = -2q_1 x a_s \Omega / (1 + k a_s^2) \omega_o F. \quad (23)$$

Combining (22) and (23), one finds that

$$V = 2q_1 k_2 x A_o E_1 \cdot a_s^2 \Omega / (1 + k a_s^2) \cdot \omega_o F \quad (24)$$

i.e.,

$$V = 2Q_L R_L k_2 x A_o^2 \left( \frac{E_1}{E} \right) \cdot \Omega a_s^2 / (1 + k a_s^2)$$

or

$$V = k (E_1/E) \Omega \cdot \frac{a_s^2/\gamma}{(1 + k a_s^2)}. \quad (25)$$

Equation (25) indicates that the output of the mixer is proportional to the carrier offset  $\Omega$ . Note that if the carrier strength is small, the discriminator characteristic will be a linear one except on the verge of the synchronization range, where  $a_s$  will decrease, resulting in a little bit of stooping at the end of the discriminator characteristics. However, a further improvement in the linearity of the discriminator characteristic can be had by increasing the sensitivity of the tracking bias generator ( $\gamma$ ). Experimental results are shown in Fig. 5, which is seen to agree well with the theoretical prediction.

##### B. Unlocked Behavior

Assuming that the center frequency of the modulating signal is equal to the free-running frequency of the oscilla-

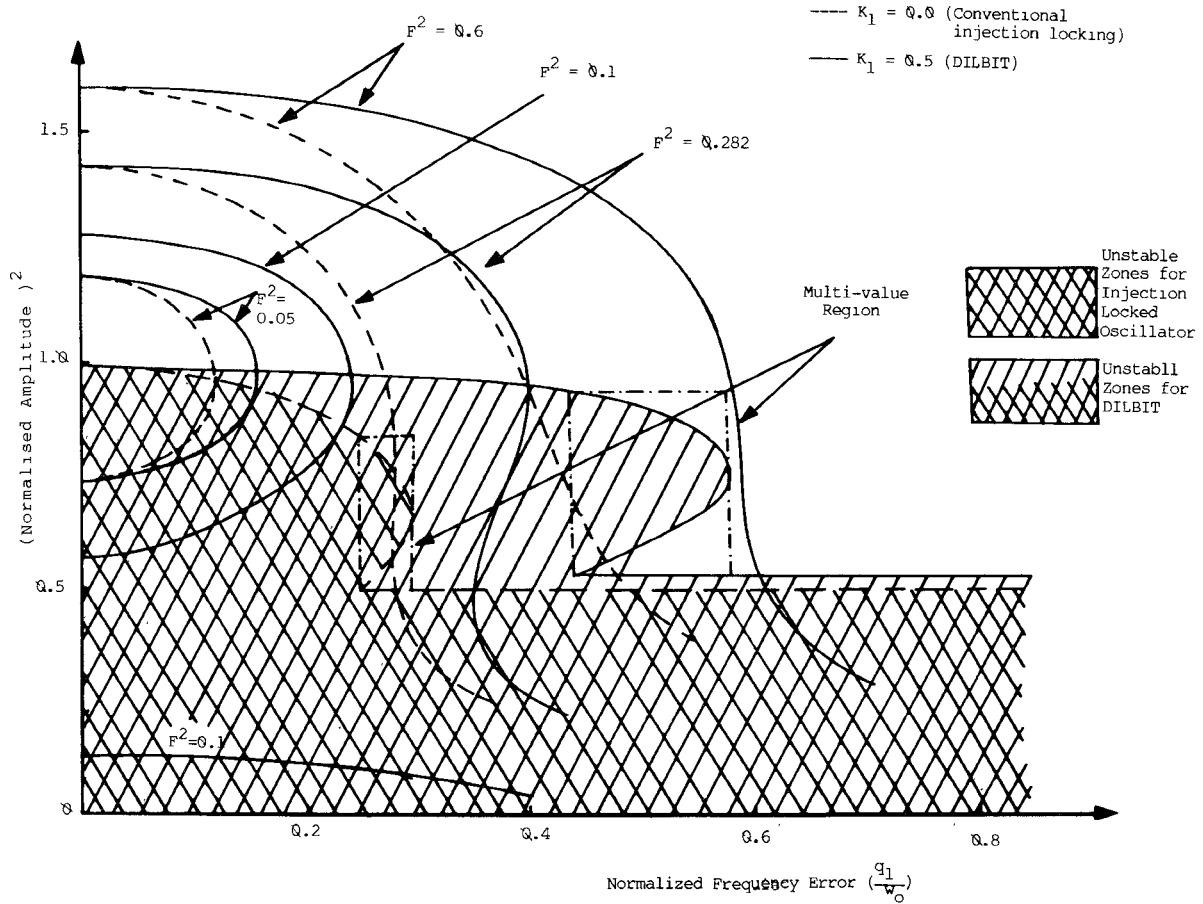


Fig. 4. The variation of the square of the normalized amplitude with the normalized frequency error.

tor, we can rewrite (13') as

$$\frac{d\varphi}{dt} = \frac{d\theta}{dt} - K_1 \sin \varphi \quad (26)$$

where

$$K_1 = \frac{\omega_o}{2q_1x} \frac{F}{a} (1 + ka^2) \quad (27)$$

and

$$\varphi = \theta - \Psi.$$

To proceed further with the calculation, we assume two things, viz., (i) the level of synchronization is low so that  $K_1$  can be treated as constant and (ii) the loop response time is much less than the time period of the modulating signal so that quasi-static operation may be assumed. Thus, putting  $\Omega_1 = d\theta/dt$ , the solution of (26) under the unlocked condition ( $\Omega_1 > K_1$ ) can be shown to be given by

$$\frac{d\varphi}{dt} = K_1 \sqrt{x_1^2 - 1} + 2K_1 \sqrt{x_1^2 - 1} \sum_{n=1}^{\infty} (-1)^n r^n \cos n(2\beta - \beta_o) \quad (28)$$

where

$$x_1 = \Omega_1 / K_1$$

$$r = x_1 - \sqrt{x_1^2 - 1}$$

$$2\beta = K_1 \sqrt{x_1^2 - 1} (t + t_o)$$

$$\beta_o = \arctan(\sqrt{x_1^2 - 1})$$

and  $t_o$  is an arbitrary constant. Therefore, the average frequency of the oscillator during the unlocked state is given by

$$\omega_{av} = \omega_1 + \left( \frac{d\Psi}{dt} \right)_{av}$$

$$= \omega_1 + \left( \Omega_1 - \sqrt{\Omega_1^2 - K_1^2} \right). \quad (29)$$

The corresponding output of the mixer/detector during the unlocked state is shown to be given by

$$V = k_2 A_2 E_1 a_s \left( x_1 - \frac{x_1^2 - 1}{x_1 + \cos(2\beta - \beta_o)} \right). \quad (30)$$

To explain the characteristics shown in Fig. 6(b), let us note that  $\Omega_1$  varies slowly with time. During the upperside of the modulating cycle, when the instantaneous frequency  $\Omega_1$  goes beyond  $K_1$ , the oscillator will slip cycles. In addition, spikes will appear, the nature of which is predicted by the relation (30). On the positive peak of the demodulated output waveform, the number of such spikes will depend on (i) the interval of time the instantaneous frequency deviation spends over  $K_1$ , (ii) the total loop

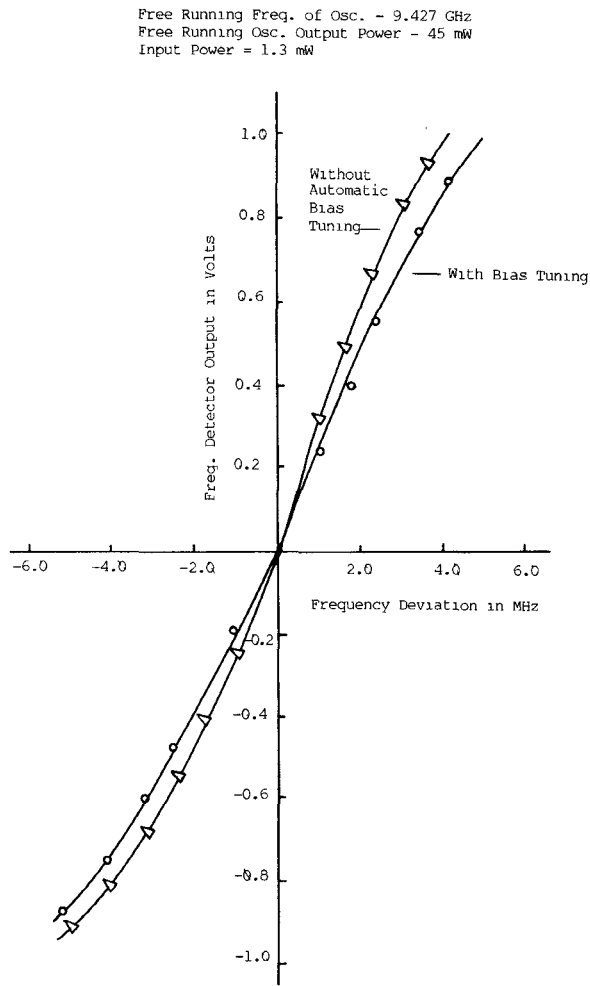


Fig. 5. The variation of frequency detector output with frequency deviation.

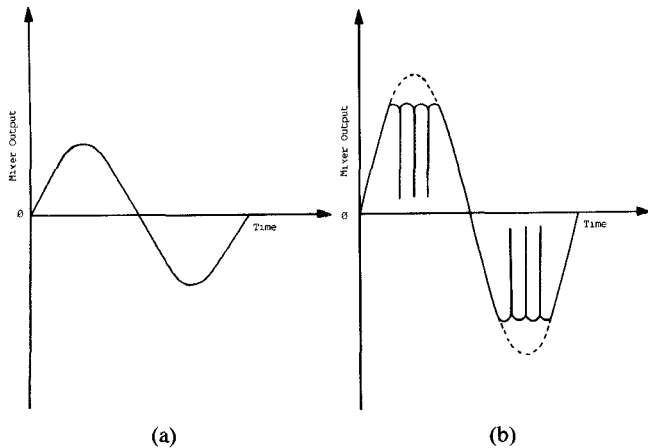


Fig. 6. Detector output for (a) small frequency deviation and (b) large frequency deviation.

gain, and (iii) the value of the modulating frequency. The explanation for the spikes appearing on the negative side of the demodulated output (Fig. 6(b)) is similar.

### C. Dynamic

To draw the dynamic discriminator characteristic, we consider the response of the oscillator to a monotone FM

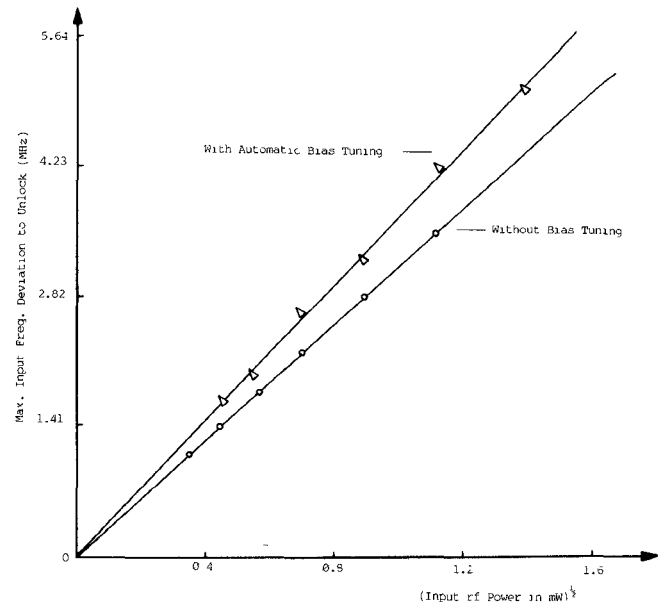


Fig. 7. The maximum frequency deviation to unlock versus square root of input RF power.

signal. We assume that the center frequency of the FM signal is equal to the free-running frequency of the oscillator. In such a situation, if the input frequency deviation is small, the oscillator will faithfully follow the input modulation, and the output of the mixer will be a replica of the input modulation, as shown in Fig. 6(a). Here we may think of quasi-static operation of the system. Thus, when the maximum instantaneous frequency deviation exceeds the hold-in range of the system, the local oscillator will slip cycles, and the output of the mixer will appear as shown in Fig. 6(b). Thus, the maximum frequency deviation to unlock versus the input is an important datum. An experimental characteristic depicting this is shown in Fig. 7. Note that Fig. 7 is a dynamic version of Fig. 3. To explain this, we refer to (26) and note that during the process of unlocking, the output becomes distorted and as such a solution of (26) is required for the case when

$$\frac{d\theta}{dt} = \Delta \cos \omega_m t \quad (31)$$

where  $\Delta$  is the maximum radian frequency deviation of the modulating signal and  $\omega_m$  is the angular frequency of the modulating signal.

Since an exact solution is not possible, we derive an approximate solution through successive approximation by replacing  $\sin \varphi$  by  $\varphi - \varphi^3/6$ . The approximate solution is shown to be given by

$$\varphi \approx \left[ \frac{\Delta}{K_1} + \frac{1}{8} \left( \frac{\Delta}{K_1} \right)^3 \right] \cos \omega_m t + \frac{1}{x_1} \left( \frac{\Delta}{K_1} \right)^3 \cdot \cos 3\omega_m t. \quad (32)$$

Now the system will lose lock when  $\varphi$  becomes  $\pi/2$ . Therefore, the maximum value of  $(\Delta/K)$  calculated from (32) comes out to be nearly 1.25, whereas the value of  $\Delta/K$  deduced from Fig. 3 and Fig. 7 is 1.22. The discrepancy is attributed to the calibration error of the FM generator.

### D. Remarks

To compare the performance of the present discriminator with that of the existing open-loop discriminators, we define a parameter called the figure of merit (FOM) as

$$\text{FOM} = \frac{\text{bandwidth} \times \text{sensitivity of the discriminator}}{\text{input power}}$$

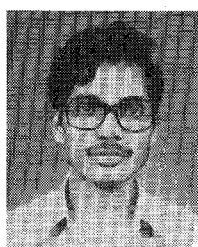
Referring to the Fig. 5, the figure of merit of DILBIT is seen to be 1.269, whereas that for the one proposed by Peebles and Green [4] is found to be about 0.05. That is, the FOM of the present discriminator is about 25 times better than that of [4]. However, when very large bandwidth is required, the use of DILBIT is not recommended. Noise and interference responses of the DILBIT are under investigation and will be reported in a future communication.

### ACKNOWLEDGMENT

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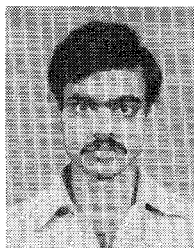
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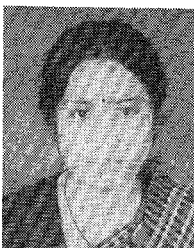
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